Millikan Oil Drop

Sreela Das

Tuesday 5th May, 2015

Week: 5
Date Performed: March 10, 2015
Partners: Alvin Modin, Lauren Hirai
Class: Intermediate Experimental Physics II

Objective

The goal of this experiment was:

- To experimentally demonstrate that charge is quantized i.e. charge is only found in discrete multiples of a certain smallest unit.
- To determine the value of this smallest discrete unit of charge.

Theory

This lab excercise for this experiment involves measuring and recording the time taken by an oil drop to first fall under gravity and viscous force of air, and then rise under force due to an applied Voltage. Since for droplets as small as those used in this experiment, terminal velocity is acheived in a matter of milliseconds, the net force can be set to zero and the viscous force set equal to the weight of the droplet giving us the following force equations for the falling drop:

$$\sum F = F_{visc} - mg = 0$$

where

$$F_{visc} = 6\pi\eta R v_f$$

R is the Radius of the oil drop,
v_f is the fall velocity of the drop, which is equivalent to the terminal velocity of the drop, and
η is the viscosity of air
This equation can be rearranged to give us an equation for the radius of the droplet in terms of the fall velocity of the droplet (which can be found empirically)

$$R = \frac{mg}{6\pi\eta v_f} = \frac{\frac{4}{3}\pi R^3 \rho g}{6\pi\eta v_f} = \frac{2R^3\rho g}{9\eta v_f} \implies R = \sqrt{\frac{9\eta v_f}{2\rho g}}$$

For droplets of radii of order comparable to the mean free path of air molecules (such as those in this experiment), Stokes' Law needs to be corrected for the effective viscosity of air which is given by:

$$\eta_{eff} = \eta\left(\frac{1}{1+\frac{b}{pR}}\right)$$

where b is a constant, p the atmospheric pressure, and R the radius of the drop. This results in a quadratic equation for the radius of the drop that solves to give:

$$R = \sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta v_f}{2\rho g}} - \frac{b}{2p}$$

Using the force equations for the system with the drop moving upwards, we get:

$$\sum F = F_E - F_{visc} - mg = 0$$

where

$$F_{visc} = 6\pi\eta R v_r \quad and \quad F_E = qE = q\frac{V}{d}$$

where v_r is the rise velocity of the drop, which is equivalent to the terminal velocity of the drop,
q is the charge on the drop,
E is the electric field under which the drop is moving,
V is the voltage applied (which creates the electric field), and
d is the plate separation or the thickness of the plate spacer
Here we can see that

$$q\frac{V}{d} = 6\pi\eta R v_r + mg = 6\pi\eta R v_r + 6\pi\eta R v_f = \frac{6\pi\eta R v_r v_f + 6\pi\eta R v_f v_f}{v_f} = \frac{(v_r + v_f)}{v_f} mg = q\frac{V}{d}$$

and so:

$$q = mg\frac{(v_r + v_f)}{v_f}\frac{d}{V} = \frac{4\pi}{3}R^3 \rho g \frac{(v_r + v_f)}{v_f}\frac{d}{V} = \frac{4\pi}{3}\frac{(v_r + v_f)}{v_f}\frac{d}{V}\left(\frac{9\eta_{eff} v_f}{2\rho g}\right)^{3/2} \rho g$$

$$q = \frac{4\pi d}{3V}\frac{(v_r + v_f)}{v_f}\left[\frac{9v_f}{2}\right]^{3/2}\left[\frac{\eta}{1+\frac{b}{pR}}\right]^{3/2}\left[\frac{1}{\rho g}\right]^{1/2} = \frac{4\pi d}{3V}(v_r + v_f)\left[\frac{9}{2}\right]^{3/2}\left[\frac{\eta}{1+\frac{b}{pR}}\right]^{3/2}\left[\frac{v_f}{\rho g}\right]^{1/2}$$

Procedure

Setup and Initial Measurements

1. The room was made as dark as possible in order to allow the droplets to be clearly visible against the screen.
2. The spacer plate was removed and its thickness recorded.
3. The capacitor plates and the spacer plate were cleaned to ensure unobstructed passage of the oil into the droplet viewing chamber.
4. The filament was adjusted until the wire in the reticle was in focus
5. the high voltage DC power supply was connected to the plate voltage connectors using banana plugs and the voltage delivered to the capacitor plates was measured using the digital multimeter.
6. The multimeter was connected to the thermistor connectors to measure the resistance of the thermistor. This value was used to match to a temperature on the thermistor conversion table to determine the temperature inside the droplet viewing chamber.

Data Collection

1. The ionization source lever was set to the Spray Droplet Position.
2. Droplets of a non-volatile oil of known density were introduced into the droplet viewing chamber.
3. Once the drops were visible through the viewing scope, the ionization source lever was returned to its off position.

4. From these drops, a drop was selected for reasonably slow speed and charge that allowed for it to be moved upwards.

5. The scope was then focused on this drop and its speed recorded over a number of runs.

6. This process was repeated for a number of droplets.

Data

The thickness of the spacer plate was found to be: 0.7 cm
Supplied Voltage was: 501.4 V
The resistance of the thermistor was recorded at: 2.060 MΩ which corresponded to a temperature of 24°C
The given density of oil was: 886 kg/m^3
The Fall time was measured with no voltage applied, while the rise time was measured with an applied voltage of 501.4 V
The radius for each run of each drop was calculated using the equation in the theory section and the mean radius for the drop was used as the radius of the drop with an uncertainty given by the standard deviation of the radius for each run.

Trial 1			
Time Falling (in s)	**Time Rising** (in s)	**Radius of Drop** (in 10^{-7}m)	**Charge on Drop** (in 10^{-19}C)
26.4	11.2	5.944	3.581
28.3	11.4	5.741	3.347
26.5	11.8	5.933	3.442
26.4	10.9	5.944	3.650
27.2	12.0	5.856	3.332

Radius of drop: $0.59 \pm 0.01 \mu$m
Charge on drop: $3.47 \pm 0.1 \times 10^{-19}$C

Trial 2			
Time Falling (in s)	**Time Rising** (in s)	**Radius of Drop** (in 10^{-7}m)	**Charge on Drop** (in 10^{-19}C)
29.4	19.0	5.633	2.312
27.1	20.5	5.867	2.381
26.9	20.9	5.889	2.372
28.3	20.0	5.741	2.321
28.0	20.4	5.772	2.317
29.5	17.9	5.623	2.391

Radius of drop: $0.58 \pm 0.01 \mu$m
Charge on drop: $2.34 \pm 0.04 \times 10^{-19}$C

Trial 3			
Time Falling (in s)	**Time Rising** (in s)	**Radius of Drop** (in 10^{-7}m)	**Charge on Drop** (in 10^{-19}C)
17.9	11.8	7.219	4.808
16.3	12.2	7.565	5.136
17.4	13.2	7.322	4.621
16.2	13.1	7.588	4.963

Radius of drop: $0.74 \pm 0.02\mu$m

Charge on drop: $4.88 \pm 0.2 \times 10^{-19}$C

Trial 4			
Time Falling (in s)	**Time Rising** (in s)	**Radius of Drop** (in 10^{-7}m)	**Charge on Drop** (in 10^{-19}C)
28.1	22.5	5.762	2.184
29.0	24.7	5.672	2.014
30.9	21.2	5.495	2.070

Radius of drop: $0.56 \pm 0.01\mu$m

Charge on drop: $2.09 \pm 0.09 \times 10^{-19}$C

Trial 5			
Time Falling (in s)	**Time Rising** (in s)	**Radius of Drop** (in 10^{-7}m)	**Charge on Drop** (in 10^{-19}C)
16.8	11.8	7.452	5.092
17.1	13.7	7.386	4.600
17.3	12.0	7.343	4.909
16.5	14.1	7.519	4.685

Radius of drop: $0.74 \pm 0.01\mu$m

Charge on drop: $4.82 \pm 0.22 \times 10^{-19}$C

Data Analysis

For an initial analysis of the data distribution, the Radius was plotted against the fall velocity making it easier to see that we had multiple values of recorded velocity for the same drop giving us different values of radius for the drop. This can be seen below where the different runs are marked with different colors. A similar plot for the charge gives us multiple values of charge for different values of fall velocity and rise velocity as well.

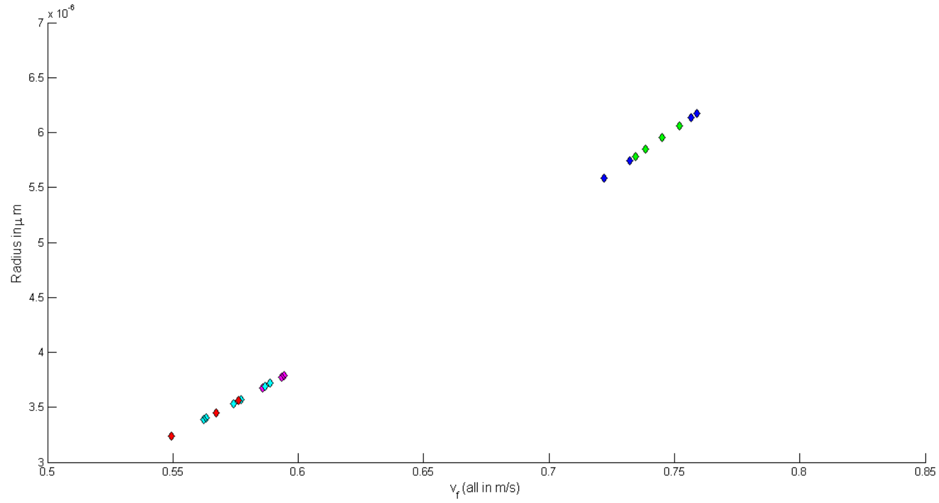

Figure 1: The Radius plotted against Fall Velocities

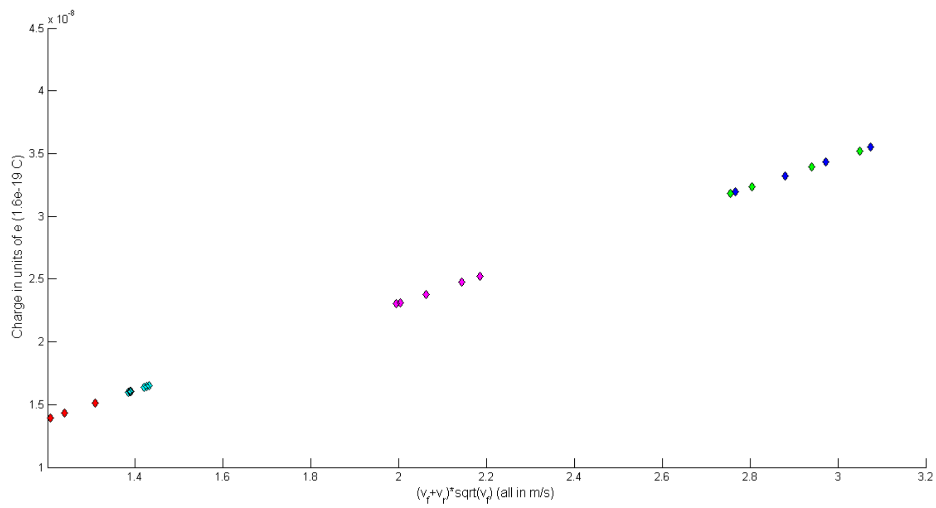

Figure 2: The Charge plotted against the velocity coefficient

The difference in fall and rise velocities over the same runs can be attributed to latency in recording the time and stopping/starting the stopwatch. The radius and charge were averaged for the different values of fall and rise velocity over the runs for the same drop. This resulted in the plots below, which show the consecutive runs in alternating colors and the average radius and charge respectively in red with the range of uncertainty shown be the error bars.

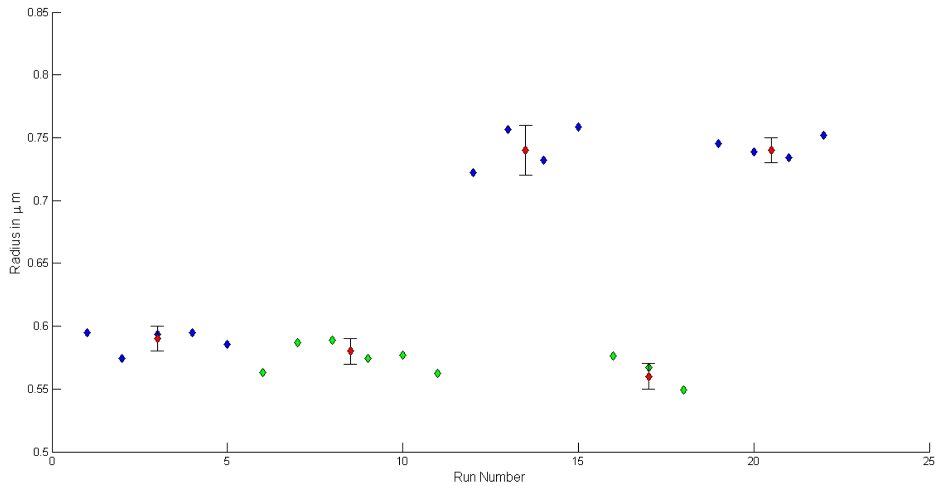

Figure 3: The Radius distribution over runs

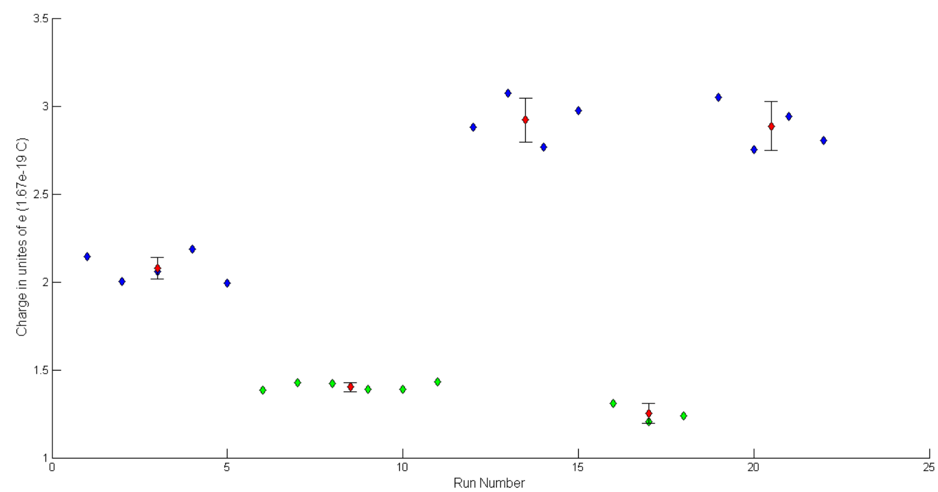

Figure 4: The Charge distribution over runs

Arranging the values of charge per drop obtained above, we get:

Charge on Drop $(\times 10^{-19}C)$	**Charge in units of calculated** q_f	**Charge in units of accepted** q_f
2.09 ± 0.09	2e	1.25e
2.34 ± 0.04	2e	1.40e
3.47 ± 0.10	3e	2.08e
4.82 ± 0.22	4e	2.88e
4.88 ± 0.20	4e	2.92e

Giving a value of fundamental charge $q_f = 1.158 \pm 0.22 \times 10^{-19}$C

Error Analysis

Since the mean over the different trials was used for the radius and charge on the drop, the uncertainty in the measurements was calculated using the standard deviation of the values for the different runs of the same drop which is given by:

$$S_N = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

The uncertainty in the final value of fundamental charge was calculated using the error propagation of weighted averages where the weight for each value was given by

$$w_i = \frac{S_i}{x_i}$$

$$\delta_{q_{fundamental}} = \frac{\sum_{i=1}^{N} w_i^2}{(\sum_{i=1}^{N} w_i)^2}$$

From these calculations, we note that the experimental uncertainty was much greater when the charge was higher (as seen in the Data Analysis section above, this corresponds to higher velocities and shorter times). This experimental uncertainty can be attributed to an increased effect of latency in recorded the time - since the time intervals were shorter, there was a greater delay in starting and stopping the stopwatch due to time taken while recording the data.

We can see however, from the table in the Data Analysis section, that the mean values recorded for the higher-charge drops gave us a much more plausible mean value of charge, whereas the data recorded with least experimental uncertainty - lowest standard deviations - turned out to be furthest from plausible (1.25e and 1.4e, both impossible). This suggests some kind of recurring error in the lower-charge readings that was unaccounted for.

Conclusions

We notice that even for the slowest recorded rise-time (20-24 seconds), the charge on the drop does not fall below a certain minimum. Theoretically, this minimum gives us the fundamental charge and all charges greater than that are integral multiples of it. We were unable to experimentally verify this accurately, but the difference between the accepted value of fundamental charge and the value that we calculated experimentally was of the order of our calculated uncertainty (although not within).

Comparing our results to the expected results tells us that there was a recurring error in our measurements for the drops carrying smaller charge - uncertainty for this part of the experiment was underestimated while uncertainty in the measurements for drops carrying greater charges was overestimated due to the more scattered distribution of data.

These uncertainties could have been reduced by taking more runs of oil drops, providing more data to average over. A stopwatch with the "lap" capabilities could have allowed for more accuracy while recording our data and would have mitigated most of the latency error.

We were able to obtain an experimental value for fundamental charge although this value did not place the accepted value within the range of uncertainty. Using only the 3 drops with higher charge on them, we could obtain an experimental value very close to the accepted value of fundamental charge (as pointed out in the Data Analysis, Trial 1 had q = 2.08e, and trials 3 and 5 had q = 2.88e, 2.92e).